

ON CALCULATION OF MICROLENSING LIGHT CURVE BY GRAVITATIONAL LENS CAUSTIC

Mikhail B. Bogdanov

Saratov State University, Bolshaya Kazachiya st., 112-a,
410026 Saratov, Russia

Abstract. For an analysis of microlensing observational data in case of binary gravitational lenses as well as for an interpretation of observations of high magnification events in multiple images of a lensed quasar it is necessary to calculate for a given source the microlensing light curve by a fold caustic. This problem comes to the numerical calculation of a singular integral. We formulated the sufficient condition of a convergence of the integral sum for this singular integral. The strictly approach to the problem of a comparison of model results with the unequally sampled observational data consists in calculation of the model light curve in equidistant points of the canonical dissection of the integration segment and a following interpolation of its values at the moments of observations.

The high angular resolution observations in optical and infrared wave bands are of great importance for the solution of many problems of astronomy and astrophysics: the search for close binaries, the measurement of angular diameters of stars, the investigation of brightness distributions across the stellar disks, the study of a fine spatial structure of active galactic nuclei (AGN). A new tool to achieving of a high angular resolution has been realized recently – the observations of gravitational microlensing.

In some microlensing events observed by MACHO, OGLE, and PLANET collaborations perhaps there are the binary gravitational lenses. For these lenses the character of the gravitational potential creates caustics in the source plane. The crossing of a caustic leads to sharp variation of a flux received from the lensed source. The character of this variation is depend on the source angular structure. Thus the observed microlensing light curve contains an information about the source brightness distribution (Albrow *et al.*, 1999; Gaudi and Gould, 1999; Bogdanov and Cherepashchuk, 2000).

The high magnification events in fluxes received from multiple images of a lensed quasar due to gravitational microlensing by stars in a lens galaxy are connected also with the complex caustic system which is created by the total gravitational potential (Schneider *et al.*, 1992; Zakharov, 1997). The observations of these events allow to obtain an information about the quasar central engine – the accretion disk, which surrounds a super massive black hole in the galactic nucleus (Grieger *et al.*, 1991; Mineshige and Yonehara, 1999; Agol and Krolik, 1999; Bogdanov and Cherepashchuk, 2001). The possible angular reso-

lution that can be realized in the caustic crossing microlensing observations is measured by the value of order or less of microsecond of arc.

From the general considerations we can believe that the case of microlensing by a fold caustic is a more probable (Gaudi and Gould, 1999). The angular sizes of observed stars of the bulge of Galaxy or the Magellanic Clouds as well as the sizes of AGN's accretion disks are very small. Therefore we may neglect of a caustic curvature and regard the fold caustic as a straightforward line. In this case the picture of microlensing depends only on one-dimensional strip brightness distribution $B(x)$ in the direction of x axis that is perpendicular to caustic. It is known that $B(x)$ is connected with a two-dimensional brightness distribution $b(x, y)$ by the integral equation

$$B(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} b(\xi, y) \delta(\xi - x) d\xi dy, \quad (1)$$

where $\delta(x)$ is the Dirac function. For a circularly symmetric source the brightness depends only on the radial distance to source center r . In this case $B(x)$ is connected with $b(r)$ by the Abel's integral equation

$$B(x) = \int_x^{\infty} \frac{2b(r)r dr}{\sqrt{r^2 - x^2}}. \quad (2)$$

For real astronomical objects the strip brightness distribution is a non-negative, continuous, and smooth enough function.

If ξ is the angular distance of point with the brightness $B(\xi)$ to center of source and x is the angular distance of source center to caustic then the coefficient of amplification can be written as (Schneider *et al.*, 1992; Zakharov, 1997):

$$A(x - \xi) = A_0 + \frac{K}{\sqrt{x - \xi}} H(x - \xi), \quad (3)$$

where $H(x - \xi)$ is the Heaviside step function ($H = 0$ for negative and $H = 1$ for non-negative values of its argument), A_0 and K are certain constants for the given caustic crossing. The observed microlensing light curve $I(x)$ as a function of distance to caustic x can be expressed by the convolution integral equation:

$$I(x) = A(x) * B(x) = \int_{-\infty}^{\infty} A(x - \xi) B(\xi) d\xi. \quad (4)$$

We can admit further without a lack of community that in the expression (3) $A_0 = 0$ and $K = 1$. Far from a caustic the integrand in the expression (4) is small enough and we can consider its to be equal to zero for $\xi < x_1$. Then as it follows from equations (3) and (4) the estimation of flux value for a given

strip brightness distribution when $x = x_n$ comes to the numerical calculation of the singular integral

$$I(x_n) = \int_{x_1}^{x_n} \frac{B(\xi) d\xi}{\sqrt{x_n - \xi}}. \quad (5)$$

It is known that the numerical estimation of a singular integral requires of certain precautions. The similar problems were examined by Belotzerkovski and Lifanov (1985). In particular it can be shown that an attempt to calculate $I(x_i)$ in arbitrary spaced points x_i when $B(\xi_i)$ is given in equidistant points ξ_i leads to a large error of the result. We formulate below the sufficient condition of convergence of the integral sum for the singular integral of our type.

At first we remind the basic definitions and one important theorem.

Definition 1

Let the segment $[x_1, x_n]$ is divided by points $x_i, i = 1, 2, \dots, n$ into $n-1$ equal sub-segments of length h , and points $\xi_i, i = 1, 2, \dots, n-1$ are the middle points of these sub-segments: $\xi_i = (x_i + x_{i+1})/2$. Then they say that the sequences of points x_i and ξ_i create the canonical dissection of this segment.

Definition 2

Let a function $f(x)$ is determined in a set D , and for two arbitrary values of its argument $x_1, x_2 \in D$ is valid the inequality

$$|f(x_1) - f(x_2)| \leq k|x_1 - x_2|^\alpha,$$

where k and α are positive numbers, and $0 < \alpha \leq 1$. Then they say that $f(x)$ satisfies the Hölder's condition of power α with the coefficient k in the set D .

Theorem 1 (The mean value theorem)

Let functions $f(x)$ and $g(x)$ are both integrable in a segment $[a, b]$, and $f(x)$ is bounded $m \leq f(x) \leq M$, and sign of $g(x)$ is invariable in this segment. Then the product of these functions $f(x)g(x)$ is also integrable, and

$$\int_a^b f(x)g(x) dx = \mu \int_a^b g(x) dx,$$

where $m \leq \mu \leq M$.

The proof of this theorem can be found in textbooks on mathematical analysis.

Take into consideration these definitions and the mean value theorem we can formulated the sufficient condition of convergence of integral sum for the rectangle rule that correspond to the singular integral (5) as the following *Theorem*.

Theorem

Let a function $B(x)$ satisfies the Hölder's condition of power α in segment $[x_1, x_n]$, and sequences of points $x_i, i = 1, 2, \dots, n$ and $\xi_i, i = 1, 2, \dots, n-1$ create the canonical dissection of this segment with the step h . Then

$$\Delta = \left| \int_{x_1}^{x_n} \frac{B(\xi) d\xi}{\sqrt{x_n - \xi}} - \sum_{i=1}^{n-1} \frac{B(\xi_i)h}{\sqrt{x_n - \xi_i}} \right| \leq o(h^{\alpha+1/2}).$$

Proof

The sequence of points x_i divides according to *Definition 1* the segment $[x_1, x_n]$ into $n-1$ sub-segments of length h . In each of these sub-segments the function $B(\xi)$ is continuous and bounded, and the sign of function $g(\xi) = (x_n - \xi)^{-1/2}$ is invariable. Using *Theorem 1* and integrate of the function $g(\xi)$ in each sub-segment we can to write

$$\Delta = \left| \sum_{i=1}^{n-1} 2B(\xi_i^*) [\sqrt{x_n - x_i} - \sqrt{x_n - x_{i+1}}] - \sum_{i=1}^{n-1} \frac{B(\xi_i)h}{\sqrt{x_n - \xi_i}} \right|,$$

where ξ_i^* is a certain point in sub-segment of number i . Further we have

$$\Delta = \left| \sum_{i=1}^{n-1} \frac{2B(\xi_i^*)\sqrt{x_n - \xi_i} [\sqrt{x_n - x_i} - \sqrt{x_n - x_{i+1}}] - B(\xi_i)h}{\sqrt{x_n - \xi_i}} \right|.$$

Let we multiply and divide simultaneously the expression that closed in brackets by factor $(\sqrt{x_n - x_i} + \sqrt{x_n - x_{i+1}})$. Then we have

$$[\sqrt{x_n - x_i} - \sqrt{x_n - x_{i+1}}] = \frac{x_{i+1} - x_i}{\sqrt{x_n - x_i} + \sqrt{x_n - x_{i+1}}} \leq \sqrt{h}.$$

The last inequality is valid so far as $x_{i+1} - x_i = h$, and the denominator of this fraction is minimal when $i = n-1$.

The common denominator in the expression for Δ is also minimal when $i = n-1$, and its minimal value is equal to $\sqrt{h/2}$ according to *Definition 1*. Thus we have restriction

$$\Delta \leq \left| \sum_{i=1}^{n-1} \frac{2B(\xi_i^*)\sqrt{h/2}\sqrt{h} - B(\xi_i)h}{\sqrt{h/2}} \right| \leq \sum_{i=1}^{n-1} \sqrt{2h} \left| \sqrt{2}B(\xi_i^*) - B(\xi_i) \right|.$$

Take into account *Definition 2* we have finally $\Delta \leq o(h^{\alpha+1/2})$. The *Theorem* is proved.

Remark 1

The Hölder's condition is a more strong restriction on the behavior of a function in comparison with a continuity. If the Hölder's condition is valid then the function is continuous. The opposite statement is generally wrong. It is obvious that for implementation of the Hölder's condition when $\alpha = 1$ sufficiently to demand of bounded first differential of the function. This demand is fulfil for strip brightness distributions $B(x)$ of real astronomical objects.

Remark 2

For increase of precision of the integral sum calculation in the same sequence of points x_i the number of points ξ_i can be multiplied by factor $k > 1$, where k is entire. The new sequence $\tilde{\xi}_i$ can determined as $\tilde{\xi}_i = x_1 + h(i - 1/2)/k, i = 1, 2, \dots, k(n - 1)$. This case is equivalent to the estimation of the singular integral in a new sequence of points $\tilde{x}_i = x_1 + ih/k, i = 1, 2, \dots, k(n - 1)$. The new points \tilde{x}_i are partly coincide with points of the previous sequence x_i and partly located between them. It is clear that the new sequences \tilde{x}_i and $\tilde{\xi}_i$ also create the canonical dissection of segment $[x_1, x_n]$.

Remark 3

The observed values of the flux in microlensing observations usually have unequal sampling intervals. Therefore the strictly approach to the problem of comparison of model results with the observational data consists in the calculation of values of the model microlensing light curve in equidistant points of the canonical dissection of the integration segment and following interpolation of them at the moments of observations.

Author is grateful to A.F.Zakharov for the attraction of his attention to this problem.

References

- Agol E., Krolik J.: 1999, *Astrophys. J.* **524**, 49.
- Albrow M.D., Beaulieu J.-P., Caldwell J.A.R. et al.: 1999, *Astrophys. J.* **512**, 672.
- Belotzerkovski S.M., Lifanov I.K.: 1985, *Numerical methods in singular integral equations*. Nauka, Moscow (in Russian).
- Bogdanov M.B., Cherepashchuk A.M.: 2000, *Astron. Rept.* **44**, 745.
- Bogdanov M.B., Cherepashchuk A.M.: 2001, *Astrophys. and Space Sci.*, in press.
- Gaudi S.B., Gould A.: 1999, *Astrophys. J.* **513**, 619.
- Grieger B., Kayser R., Schramm T.: 1991, *Astron. and Astrophys.* **252**, 508.
- Mineshige S., Yonehara A.: 1999, *Publ. Astron. Soc. Japan.* **51**, 497.
- Schneider P., Ehlers J., Falco E.E.: 1992, *Gravitational lenses*. Springer, Berlin.
- Zakharov A.F.: 1997, *Gravitational lenses and microlenses*. Yanus-K, Moscow (in Russian)